

constant over the range of stress concerned, and that the materials should be adequately isotropic as regards deformation under stress.

Provided these several conditions are adequately met, the distortions under pressure should be proportional to one another and should be in the inverse ratio of the elastic moduli of the two materials.

It is of interest to note that the distribution of pressure along the gap may be influenced by the manner in which the coefficient of viscosity of the fluid depends on pressure. For a given total flow of fluid and a given width of gap, the pressure gradient at any point will be proportional to the local value of the coefficient of viscosity. Since in turn the precise form of the distortion of the boundaries of the cylinder and piston depend upon the distribution of pressure along the gap, it is clear that the resulting distortion will depend in a complicated manner on a number of interacting factors. Under the similarity condition, however, all these complications are avoided.

Under the conditions postulated the distortions may be expressed in the form

$$\Delta(\alpha, P) = \alpha f(P); \Delta(\beta, P) = \beta f(P) \quad (1.42)$$

where  $f(P)$  is an unknown function of the pressure but is the same function in the case of both assemblies, and  $\alpha$  and  $\beta$  are, as before, two constants inversely proportional to the elastic moduli of the two materials. In these circumstances equation (1.41) may be written

$$\frac{A_P}{B_P} = \frac{A_0}{B_0} [1 + (\alpha - \beta)f(P)] \quad (1.43)$$

The experimental procedure is now firstly to determine the quantity  $(\alpha - \beta)f(P)$  by direct balancing over the range of pressure concerned, and secondly to determine the ratio  $\alpha/\beta$  either by direct measurements of the elastic moduli or by any other method which may be available. The data provided by these two procedures clearly enables the distortion of each assembly to be derived in absolute terms.

In the present series of experiments the two materials employed are a very hard tool steel and a special bronze of high tensile strength, the elastic modulus of which was known to be about two-thirds that of steel which is a convenient ratio from the experimental point of view. Measurements of the ratio  $\alpha/\beta$  have been made by two substantially different methods. In the first place direct measurements of the elastic constants have been carried out in the Engineering Section of the Physics Division using the customary static methods for determining these quantities over a wide range of stress. The stress-strain relationships were determined both for the modulus of rigidity, using the torsion method, and for Young's modulus, using the standard extensometers employed at the N.P.L. The values were also checked by the ultrasonic wave velocity method, where the stresses were considerably smaller

than those involved in the pressure experiments. The ultrasonic measurements, however, provided a useful check of the degree of isotropy of the materials, and it may be noted that wave measurements in three directions at right-angles to one another gave practically indistinguishable results. One difficulty in the determination of  $\alpha/\beta$  by the direct measurement of elastic moduli lies in the choice of samples of material for the construction of the necessary test pieces. The component parts of a piston-cylinder assembly are not themselves ideally shaped for extensometer measurements, and it is therefore important that test specimens should be chosen as far as possible from the same batch of material as that from which the assemblies themselves were constructed. If this is not possible care must be taken to ensure that the constitution of the material, and any heat treatments which may be necessary, are identical in both cases. Certain discrepancies in the elastic constant measurements which were encountered in the early stages of the work are thought to have been due to uncertainties in composition or heat treatment of some of the samples of steel which were employed.

In order to avoid the slight hazard attaching to the sampling of test pieces, and in an attempt to determine the ratio  $\alpha/\beta$  on the actual assemblies themselves, an independent method has also been developed. This method makes use of precisely the same principle of similarity as has been discussed above. The general procedure is to measure the rate of flow of the pressure transmitting fluid through the gap between piston and cylinder in the case of each assembly at a series of corresponding pressures. Bearing in mind that the rate of flow for a given pressure distribution, and therefore a given distribution of the coefficient of viscosity, is a function of the width of the gap and the manner in which this is distributed along the axial length, a detailed examination shows that these measurements provide sufficient data for the calculation of the ratio of the distortions of the two gaps in terms of their original values at zero pressure. This method has given agreement as regards the value of  $\alpha/\beta$  to within about 2 per cent with the value afforded by the direct measurements of the elastic constants, the mean being very close to 1.50.

Reference also needs to be made to the possible effects of inaccuracies or non-uniformity in the dimensions of the pistons and cylinders. The precise application of the principle of similarity as outlined above was at first thought to necessitate very accurate similarity between the contours of the components under the condition of zero pressure, and in the earliest measurements great care was taken to conform with this requirement. This proved to be possible owing largely to improvements which have been made in the Metrology Division of the Laboratory in the measurement of the form and diameters of cylinder bores (Taylerson 1955). In view of the resulting difficulties in construction, however, it was decided to investigate directly to what